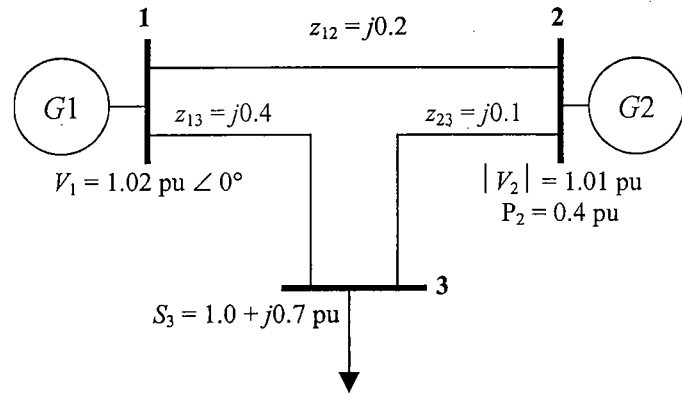


1. From the one-line diagram of a three-bus system shown above, find the bus admittance matrix. (20 pts)



$$y_{12} = -j5.0$$

$$y_{13} = -j2.5$$

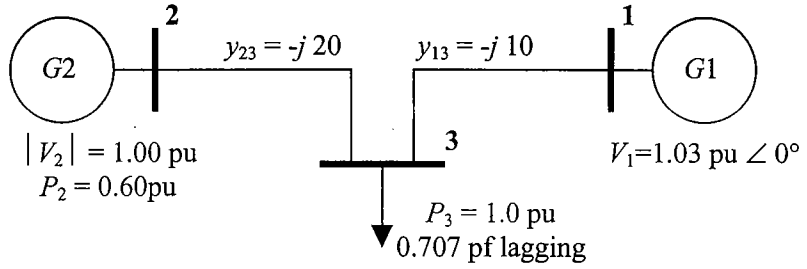
$$y_{32} = -j10.0$$

$$Y_{BUS} = j \begin{bmatrix} -7.5 & 5 & 2.5 \\ 5 & -15 & 10 \\ 2.5 & 10 & -12.5 \end{bmatrix}$$

$$y_{LOAD} = \left( \frac{S}{V} \right)^* = \frac{1.0 - j0.7}{1.0}$$

$$Y'_{BUS} = \begin{bmatrix} -j7.5 & j5.0 & j2.5 \\ j5.0 & -j15 & j10.0 \\ j2.5 & j10.0 & 1 - j13.2 \end{bmatrix}$$

2. For the network shown above, find new voltage angles and magnitudes of buses #2 and #3 after ~~two~~ (2) iteration [k=2] using the Fast Decoupled power flow method (assume flat starting conditions,  $1 \angle 0$ ). (40 pts)



INITIAL COND.

$$\begin{bmatrix} \delta_2^0 \\ \delta_3^0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} V_3^0 \end{bmatrix} = \begin{bmatrix} 1.0 \end{bmatrix}$$

$$Y_{BUS} = j \begin{bmatrix} -10 & 0 & 10 \\ 0 & -20 & 20 \\ 10 & 20 & -30 \end{bmatrix}; \quad -B' = \begin{bmatrix} 20 & -20 \\ -20 & 30 \end{bmatrix}; \quad -B'' = \begin{bmatrix} 30 \end{bmatrix}$$

$$[-B']^{-1} = \begin{bmatrix} 0.15 & 0.1 \\ 0.1 & 0.1 \end{bmatrix}; \quad [-B'']^{-1} = \begin{bmatrix} 1/30 \end{bmatrix}$$

$$P_2 = (1.0)(1.0)(20) \cos(-90 + 0 - 0) + (1.0)(1.0)(20) \cos(90 + 0 - 0) = 0$$

$$P_3 = (1.0)(1.0)(30) \cos(-90 + 0 - 0) + (1.0)(1.0)(20) \cos(90 + 0 - 0) + (1.03)(1.0)(10) \cos(90 + 0 - 0) = 0$$

$$-Q_3 = (1.0)(1.0)(30) \sin(-90) + (1.0)(1.0)(20) \sin(90 + 0 - 0) + (1.03)(1.0)(10) \sin(90 + 0 - 0) = -30 + 20 + 10.3 = 0.3$$

$$\begin{bmatrix} \Delta \delta_2 \\ \Delta \delta_3 \end{bmatrix} = \begin{bmatrix} 0.15 & 0.1 \\ 0.1 & 0.1 \end{bmatrix} \begin{bmatrix} \frac{0.6 - 0.0}{1.0} \\ \frac{-1.0 - 0.0}{1.0} \end{bmatrix} = \begin{bmatrix} -0.01 \\ -0.04 \end{bmatrix} \quad \begin{matrix} \delta_2^{[1]} = -0.01 \\ \delta_3^{[1]} = -0.04 \end{matrix}$$

$$\begin{bmatrix} \Delta V_3 \end{bmatrix} = \begin{bmatrix} 1/30 \end{bmatrix} \begin{bmatrix} \frac{-1.0 - 0.3}{1.0} \end{bmatrix} = \begin{bmatrix} -0.0233 \end{bmatrix} \quad V_3^{[1]} = 0.977$$

$$P_2 = (1.0)^2(20) \cos(-90) + (1.0)(0.977)(20) (\cos(+90 - 0.01 + -0.04)) = 0.586$$

$$P_3 = (0.977)^2(30) \cos(-90) + (1.0)(0.977)(20) \cos(+90 - 0.04 + 0.01) + (0.977)(1.03)(10) \cos(90 + 0.04 - 0.01) = 0 - 0.586 - 0.402 = -0.988$$

$$-Q_3 = (0.977)^2(30) \sin(90) + (0.977)(1.0)(20) \sin(90 - 0.04 + 0.01) + (0.977)(1.03)(10) \sin(90 - 0.04 + 0) = -28.64 + 19.53 + 10.05 = 0.94$$

$$\begin{bmatrix} \Delta \delta_2 \\ \Delta \delta_3 \end{bmatrix} = \begin{bmatrix} 0.15 & 0.1 \\ 0.1 & 0.1 \end{bmatrix} \begin{bmatrix} \frac{0.6 - 0.586}{0.977} \\ \frac{-1.0 - 0.988}{0.977} \end{bmatrix} = \begin{bmatrix} 0.00087 \\ 0.00017 \end{bmatrix}$$

$$\begin{bmatrix} \Delta V_3 \end{bmatrix} = \begin{bmatrix} 1/30 \end{bmatrix} \begin{bmatrix} \frac{-1.0 - 0.94}{0.977} \end{bmatrix} = \begin{bmatrix} -0.002 \end{bmatrix}$$

$$\delta_2^{[2]} = -0.00913$$

$$\delta_3^{[2]} = -0.0398$$

Problem Score: \_\_\_\_\_

$$V_3^{[2]} = 0.975$$

3. The generator cost functions of three generators are given by

$$C_1 = 290 + 5.0 P_1 + 0.008 P_1^2 \quad (\$/h/MW), \quad 100 \leq P_1 \leq 300$$

$$C_2 = 270 + 5.5 P_2 + 0.009 P_2^2 \quad (\$/h/MW), \quad 200 \leq P_2 \leq 350$$

$$C_3 = 300 + 4.5 P_3 + 0.007 P_3^2 \quad (\$/h/MW), \quad 175 \leq P_3 \leq 400$$

where the powers are given in MW. The total load demand is 550 MW.

- a) Find the optimal dispatch of the three generators without any limits (20 pts)  
b) What is the new dispatch if generator limits are enforced (20 pts)

If you use the method of iterations, you may start with an initial value of 6

$$a) \quad \beta_1 = 5.0 \quad \gamma_1 = 0.008 \quad P_{LOAD} = 550 \text{ MW}$$

$$\beta_2 = 5.5 \quad \gamma_2 = 0.009$$

$$\beta_3 = 4.5 \quad \gamma_3 = 0.007$$

$$\lambda(a) = \frac{550 + \frac{5}{0.016} + \frac{5.5}{0.018} + \frac{4.5}{0.014}}{\frac{1}{0.016} + \frac{1}{0.018} + \frac{1}{0.014}} = 7.861$$

$$P_1 = \frac{7.861 - 5.0}{0.016} = 178.8$$

$$P_2 = \frac{7.861 - 5.5}{0.018} = 131.1$$

$$P_3 = \frac{7.861 - 4.5}{0.014} = 240.1$$

$$b) \quad P_2 = 200 \quad \text{MEET LOWER LIMIT}$$

$$\lambda(b) = \frac{350 + \frac{5}{0.016} + \frac{4.5}{0.014}}{\frac{1}{0.016} + \frac{1}{0.014}} = 7.347$$

$$P_1 = \frac{7.347 - 5.0}{0.016} = 146.7$$

$$P_3 = \frac{7.347 - 4.5}{0.014} = 203.3$$